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RESEARCH ARTICLE

Some statistical aspects of the semi-probablistic approach (partial factoring) of the EUROCODE 7

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Abstract

The definition of the characteristic values of the soil parameters (according to EUROCODE 7) constitutes one of the most critical tasks in geotechnical calculations. The methods and the procedures related to the calculation of the characteristic values for the soil parameters with normal or lognormal distribution, related to four several cases (for statistical "known" or "unknown" parameter and for average value or extreme value), considering the cases when previous knowledges are or are not available are summarized in this paper. The confidence level concept is connected to the design value and the partial factor concepts. In addition, some interesting cases are analysed (i.e. the case of parameters that vary with depth, a case where the characteristic value of the lower estimate of the mean is greater than the upper estimate of the mean of the parameter value).

Keywords

soil parameter · characteristic value · coefficient of variation · confidence level

Acknowledgement

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1 Introduction

According to the Eurocode 7 [1], the characteristic values of geotechnical parameters shall be defined according to the results obtained from field and laboratory tests and derived values, with attention to the following details:

- geological and other background information, as e.g. data resulting from earlier construction works;
- variability of the values of the measured characteristic and further significant information, e.g. based on already existing knowledge;
- amount of the field and laboratory tests;
- type and quantity of samples;
- extension of the soil zone influencing the behavior of the geotechnical structure in the investigated ultimate limit state;
- ability of the geotechnical structure to transfer loads from a weak soil zone to a stronger one.

The standard also emphasizes that during the definition of the characteristic values, the higher coefficient of variation of the effective cohesion (c') relating to the internal frictional angle (tan ϕ') shall be taken into consideration.

The overwhelming majority of parameters used in geotechnical calculations follows a normal or lognormal distribution, and I only discussed in the following sections the statistical methodology of parameters that can be characterized by the above two types of distribution.

In the whole paper its used four cases for definition of characteristic values: for statistical "known" (I.) or "unknown" parameter (II.) and for average value (A) or extreme value (B). Specification of these cases are in Chapter 2.1.

2 The characteristic value

2.1 Characteristic value in case of a normal distribution of the parameters

In case of a normal distribution, the characteristic values (X_k) can be defined with statistical methods according to the following relationship [1]:

$$X_k = X_m \cdot (1 \pm k_n \cdot v_x) \tag{1}$$

The interpretation of the characteristic values is illustrated in Fig. 1. The factors figuring in the formula are:

- X_m is the expected value, which can unbiasly estimated by the mean of the data;
- k_n is a statistical parameter depending on the number of samples;
- ν_x is the coefficient of variation, assumed according to previous knowledges, i.e. supposed to be statistically "known" (hereinafter referred to: Case I), or calculated from measurement results, i.e. regarded statistically (previously) "unknown" (hereinafter referred to: Case II).

The coefficient of variation is the quotient of the standard deviation (S_x) and the average value:

$$\nu_x = \frac{S_x}{X_m} \tag{2}$$

The $\pm$ sign in the Eq. (1) expresses that the expression can be used for both the lower and upper estimates, resulting in symmetric results obtained for the average value.

The extension of the soil zone determining the behavior of the "geotechnical structure" at any ultimate limit state is generally much higher than the size of the soil sample or that of the zone affected by the in situ test. Consequently, the value of the dominant parameter is often identical with the average of the values related to a surface or volume of the soil. It is advisable to assume the characteristic value with a careful estimation of this average value (hereinafter referred to: A, Fig. 3). In case the behavior of the geotechnical structure is defined in the investigated ultimate limit state by the lowest or highest value of the soil characteristic, it is advisable to assume the characteristic value with a careful estimation of the lowest or highest possible value of the zone determining the behavior (hereinafter referred to: B, Fig. 3).

The standard "only" includes the following: "In case of applying statistical methods, it is appropriate to derive the characteristic value in a way that the calculated probability of the unfavorable value defining the investigated ultimate limit state be not higher than 5%" (A). In this respect, "a careful estimate of the average value means the average of a limited set of the geotechnical parameters is selected at a confidence level of 95%" (B).

Definition of the values of the k_n statistical parameter related to a confidence level of 95% in the combinations of the above cases [2]:

A-I. (average value, a statistically "known" parameter):

$$k_n = t_{\infty}^{95\%}\sqrt{\frac{1}{n}} \tag{3}$$

A-II. (average value, a statistically "unknown" parameter):

$$k_n = t_{n-1}^{95\%}\sqrt{\frac{1}{n}} \tag{4}$$

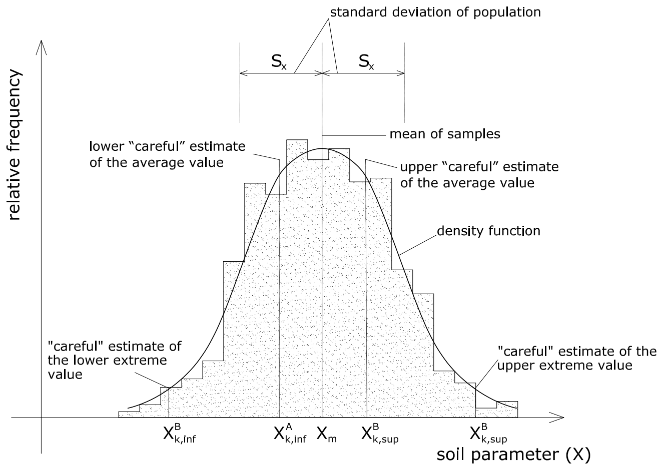

Fig. 1. Illustrative graphic for the interpretation of the characteristic value (normal distribution)

B-I. (extreme value, a "known" parameter):

$$k_n = t_{\infty}^{95\%}\sqrt{\frac{1}{n}+1} \tag{5}$$

B-II. (extreme value, an "unknown" parameter):

$$k_n = t_{n-1}^{95\%}\sqrt{\frac{1}{n}+1}. \tag{6}$$

Generally, a statistical parameter is regarded as "known", if the coefficient of variation or a real upper limit of this factor is already known, on the basis of previous data. (This definition necessarily implies that also the type of distribution of the parameter is known).

Instead of applying the rules related to the procedure performed according to the "unknown" coefficient of variation, it is advisable to use in the practice the procedure performed according to the "known" coefficient of variation, whereas the coefficient of variation is taken into consideration with a safe upper estimate.

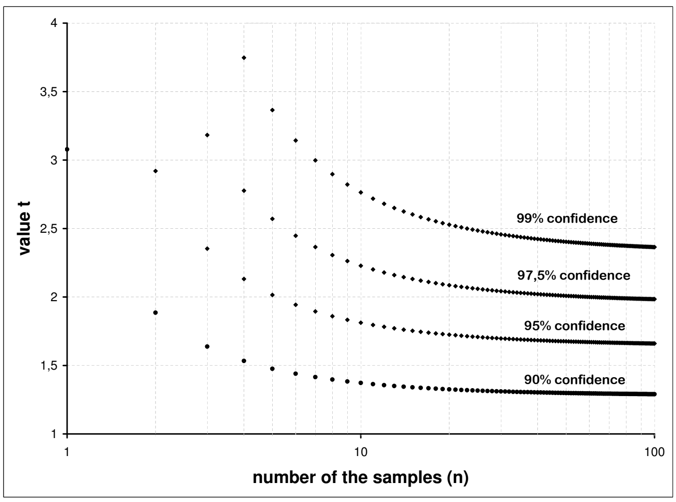

Fig. 2. Student's t values for 90; 95; 97,5 and 99% confidence levels

The (single limit type) t values of the t distribution according to Student at the confidence levels of 90, 95 and 99% can be assumed according to the diagrams of Fig. 2. At an infinite degree of freedom, the value of t at a confidence level of 95% is as follows:

$$t_{\infty}^{95\%} \approx 1,645. \tag{7}$$

Fig. 3 contains the values of $k_{n:}$ the two lower curves can be used for the average value (probability level of 50%), and the two upper curves for the extreme values (probability level of 5%). Calculation of four points on the chart in Fig. 3 can see in Appendix A. In both cases, separate curves are related to the statistically "known" and statistically "unknown" data sets. In case of a statistically "known" and "unknown" coefficient of variation, already a minor deviation becomes visible between the $k_n$values above appr. 10 samples. This deviation can be experienced in case of the extreme values above appr. 30 samples.

As a result obtained from his comparative investigations, Schneider [3] suggested to assume the characteristic value (referring to the lower estimate of the average value, indicated with A above) approximately at the value that is lower than the average value by half of the coefficient of variation:

$$X_k = X_m \cdot (1 - 0,5 \cdot \nu_x). \tag{8}$$

The main advantage of this approach is that it can also be used when there are no data available. This type of approach is being accepted and used in Switzerland since 1990, and in other countries of Europe since 1997 [3].

In the practice of geotechnics, there are in certain cases no sufficient data available for the determination of standard deviation. In such cases, a consideration of the values indicated in Tab. 1 is suggested [3–7] for the definition of the characteristic values.

The characteristic value of the soil parameters is the basis of the geotechnical calculation, especially well appliable for special problems, e.g. slope stability analyses or flood dyke failure probabilistic calculation [8].

2.2 Characteristic value in case of a lognormal distribution of the parameters

If the distribution of the parameter (X) follows a lognormal pattern, the characteristic values (X_k) should be defined according to [9] as follows:

$$X_k = e^{[Y_m - k_n^{\cdot} s_y]}, \tag{9}$$

where Y_m is the average of the $\ln(X_i)$ values:

$$Y_m = \frac{1}{n}\sum \ln(X_i). \tag{10}$$

s_y standard deviation of the lognormal distribution:

- if ν_x is known from previous data:

$$s_y = \sqrt{\ln(\nu_x^2 + 1)} \approx \nu_x \tag{11}$$

- if ν_x is not known from previous data:

$$s_y = \frac{1}{n-1}\sum (\ln X_i - Y_m)^2 \tag{12}$$

If ν_x is statistically known, the Y_m average of the parameter with lognormal distribution can also be calculated with the use of the X_m average and the s_y standard deviation:

$$Y_m = \ln(X_m) - 0,5 \cdot s_y^2. \tag{13}$$

2.3 Lower or upper estimate?

In the majority of geotechnical calculations, it is easy to decide in advance which of the characteristic values shall be used: the value defined by lower or by upper estimation, and the consideration of which of them produces the most disadvantageous situation: it represents the lowest value in case of resistance but the highest one in case of effect. It is also possible however, that a designing situation occurs, in which it cannot be decided clearly in advance which one of the estimations shall be used, and preliminary investigations are required for this purpose.

As an example, let us analyze the behavior of a U-shaped reinforced concrete structure built into granular soil, visualized in Fig. 4, from the aspect of uplift (UPL ultimate limit state). The V_{dst} hydrostatic uplift force and the G_{sz} weight of the structure that are only necessary for the investigation of uplift do not play any role in the below analysis concerning soil characteristics. The characteristic value of the frictional force arising on the lateral wall A-B of the structure, pointing downwards, and affecting both lateral walls (being the product of the force pressing the surfaces together and of the frictional coefficient) amounts to:

$$T_k = 2 \cdot E_{a,k}^{\cdot} tg\delta_k \tag{14}$$

Characteristic value of the active earth pressure:

$$E_{a,k} = \frac{H^2 \cdot \gamma_k}{2} \cdot K_a \tag{15}$$

Active earth pressure factor in Eq. (15):

$$K_a = tg^2\left(45^\circ - \frac{\varphi_k}{2}\right) \tag{16}$$

The maximum of the friction angle of the wall equals to the 2/3 of the internal angle [1]:

$$\delta_k = \frac{2}{3} \cdot \varphi_k \tag{17}$$

The $T_k$frictional force, substituting of Eq. (15), (16) and (17) is:

$$T_k = H^2 \cdot \gamma_k \cdot tg^2\left(45^\circ - \frac{\varphi_k}{2}\right) \cdot tg\left(\frac{2}{3} \cdot \varphi_k\right) \tag{18}$$

$H^2 \cdot \gamma_k$ can be regarded to be constant because the height (H) is taken at a nominal value into consideration, and the (γ_k) coefficient of variation of the weight density is considered in the practice as 0 (Tab. 1). Let us introduce the factor β for the product of the variable members of the Eq. (18):

$$\beta = tg^2\left(45^\circ - \frac{\varphi_k}{2}\right) \cdot tg\left(\frac{2}{3} \cdot \varphi_k\right) \tag{19}$$

Thus, the investigation of changes of the T_k frictional force in function of the ϕ_k frictional is simplified into the analysis of

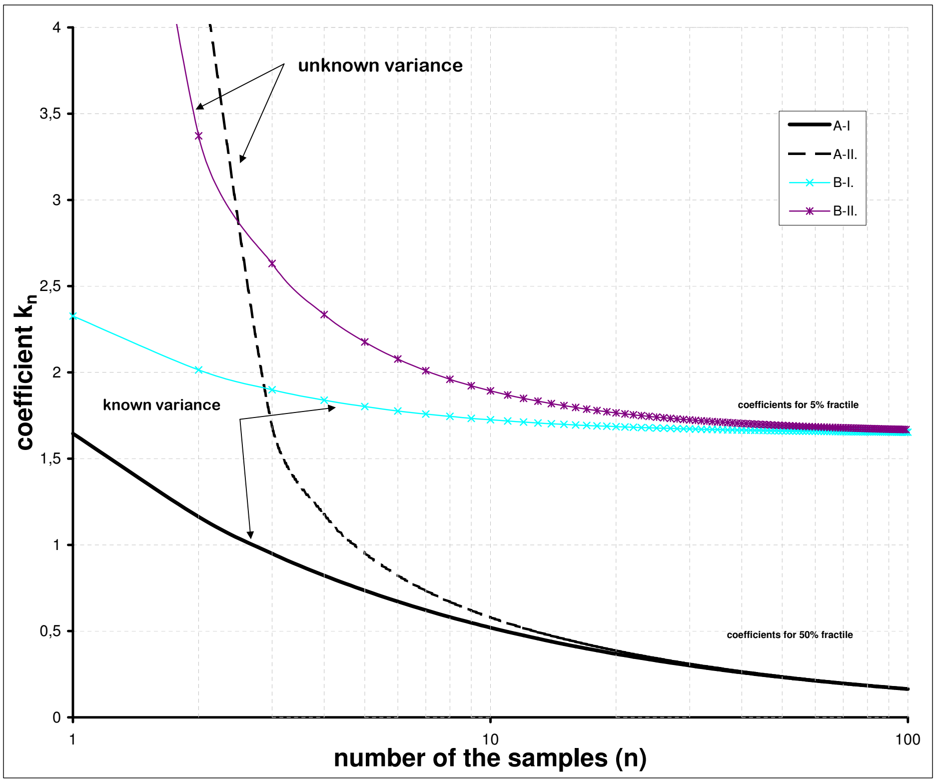

Fig. 3. Values of coefficient k_n (statistical coefficients for determining the 5 and 50 % fractile with 95 % confidence)

Tab. 1. Characteristic range and suggestible value of coefficient of variation (compiled from the data of [3–7])

Type of soil parameter	Notation	Characteristic range of coefficient of variation	Suggestible value of coefficient of variation
Effective friction angle	tan ϕ'	0,04-0,30	0,1
Effective cohesion	c'	0,3-0,6	0,4
Undrained strength	c_u	0,2-0,4	0,3
Oedometer modulus	E_s	0,2-0,7	0,4
Weight density	γ	0,01-0,1	0,05*
*The weight density is generally taken at its characteristic value into consideration (ν=0).			

coefficient β. A plotting of the $\beta(\phi_k)$ curve shows that it is increasing strictly monotonously in case of low internal frictional angles, reaching its maximum value at ϕ_k=27,3°, then decreasing above it strictly monotonously (Fig. 5). All this means that this preliminary investigation has to be performed in order to decide whether the lower or upper estimation should be used, i.e. actually which of the $\pm$ marks should be taken into consideration in Eq. (1).

3 Definition of the expected value and coefficient of variation

As it turned out above, only two parameters (the expected value and the coefficient of variation) are necessary to define the X_k characteristic value. Schneider [3] distinguishes three, basically differing possibilities providing a basis for the estimation or calculation of the X_m and ν_x values:

- there are no test results available, only previous (a priori)

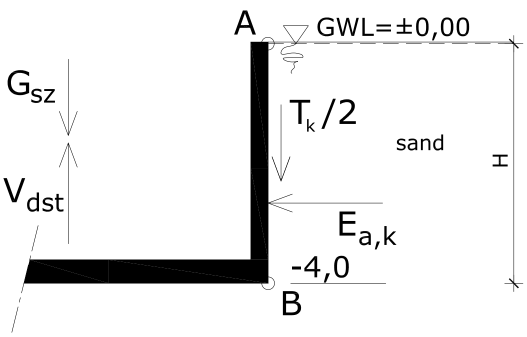

Fig. 4. Investigation of the uplift of a U-shaped reinforced concrete structure built into granular soil

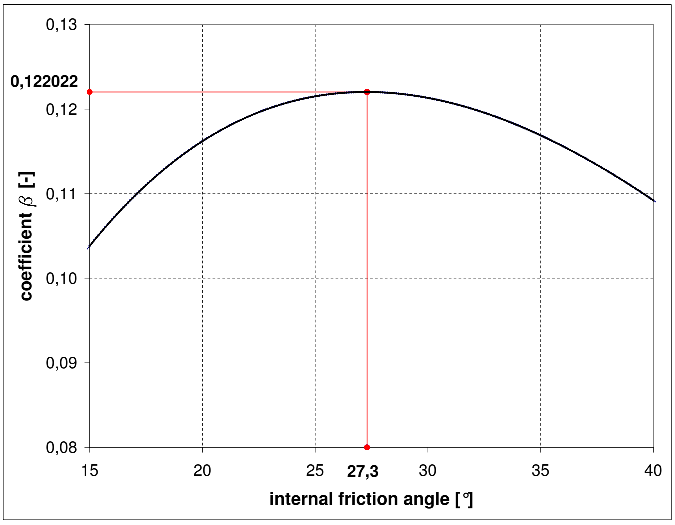

Fig. 5. Alteration of coefficient β in function of the internal frictional angle

knowledges concerning the parameter,

- there is an ample quantity of numerical test results available,
- the "combination" of the above two possibilities: beside test results, also previous knowledges (*a priori*) information is available; e.g. a compression modulus defined by means of compression test can be combined with the results obtained from a pressure sounding (CPT), beat sounding (DP), or a lapidometric test (DMT) [10].

a,) There are no test results available ($n = 0$):

$$X_m = \frac{X_{\min.} + 4X_{\text{mode}} + X_{\max}}{6} \tag{20}$$

$$\nu_x = \frac{6 \cdot (X_{\max} - X_{\min})}{X_{\min} + 4X_{\text{mode}} + X_{\max}}, \tag{21}$$

where $X_{\min}$ is the estimated minimum value, X_{mode} the value of highest probability, and $X_{\max}$ the estimated maximum value. The estimation of the values ($X_{\min}$, X_{mode} and $X_{\max}$) can be based on personal experience or judgment, documented local or regional experience as well as published tables with typical values.

All this means that $X_{\min}$ and $X_{\max}$may deviate from the expected value by a threefold of the standard deviation.

As an alternative possibility, a deviation of the values of $X_{\min}$ and $X_{\max}$ by a twofold standard deviation from the expected value can be assumed (Bond and Harris, 2008); in this case:

$$\nu_x = \frac{1,5 \cdot (X_{\max} - X_{\min})}{X_{\min} + 4X_{\text{mode}} + X_{\max}}. \tag{22}$$

b,) There are only test results available:

In this case, both the expected value and the coefficient of variation can be specified according to generally known statistical relationships.

c,) „combination": both the test results and the a priori information are available

On the basis of the Bayes theory and according to the suggestion of Tang [11], the available test results and *a priori* information can be combined in the below described way. The results thus obtained are more reliable than those obtained in the cases a.) or b.)

The *a priori* values estimated in item a.) are: X_{m1}, ν_{x1} and $S_{x1} = X_{m1} \cdot \nu_{x1}$

The results obtained from the tests are: X_{m2}, S_{x2} and ν_{x2}

$$X_{m2} = \frac{\sum X_i}{n}; \tag{23}$$

$$S_{x2} = \sqrt{\frac{\sum (X_i - X_{m2})^2}{n-1}}; \tag{24}$$

$$\upsilon_{x2} = \frac{S_{x2}}{X_{m2}}. \tag{25}$$

The combined results are:

$$X_{m3} = \frac{X_{m2} + \frac{X_{m1}}{n}\left(\frac{S_{x2}}{S_{x1}}\right)^2}{1 + \frac{1}{n}\left(\frac{S_{x2}}{S_{x1}}\right)^2}; \tag{26}$$

$$S_{x3} = \sqrt{\frac{S_{x2}^2}{n + \left(\frac{S_{x2}}{S_{x1}}\right)^2}}; \tag{27}$$

$$\nu_{x3} = \frac{S_{x3}}{X_{m3}}. \tag{28}$$

At the same time, it should be emphasized as a main condition that only the statistical processing of "suitable" results can lead to a suitable end result: both the erroneous measurement results and the unreliable a priori data or those defined with non-unambiguous conditions shall be neglected (Réth※ti, 1988.) Their filtering out may in certain cases become a harder engineering task than the statistical processing or the dimensioning of the structure (the remark "especially carefully" of the standard refers to this point).

4 Parameters that vary with depth

In some cases, the soil parameters (X) are in a definitive correlation with the subsurface depth (z) (e.g. the compression modulus grows in certain cases linearly with the increase of depth). Such correlations can be investigated by means of multi-variate statistics and calculated with the use of the below relationships [2].

In the depth z_i the value of soil parameter is X_i, the number of data is n.

The characteristic value of a parameter X that varies linearly with depth below the ground surface (z) is:

$$X_k = X_m + b \cdot (z - m_z) \pm \varepsilon_n, \tag{29}$$

where X_m is the expected value; m_z is the average of the depths z, and ε_n is the so called standard error.

Tab. 2. Partial factors of the soil parameters (γ_M), with the summary of Tables A2., A4., A16. and NA2. of standard MSZ EN 1997-1:2006 [1]

Type of soil parameter	Notation	Ultimate limit state				
		EQU	STR and GEO – Value group M1	STR and GEO – Value group M2	UPL	Slopes[b]
Effective internal friction angle[a]	$\gamma_{\phi'}$	1,25	1,0	1,25	1,25	1,35
Effective cohesion	$\gamma_{c'}$	1,25	1,0	1,25	1,25	1,35
Undrained strength	γ_{cu}	1,4	1,0	1,4	1,4	1,50
Unconfined compressive strength	γ_{qu}	1,4	1,0	1,4		1,50
Weight density	γ_{γ}	1,0	1,0	1,0		1,0
Resistance of the drawn piles	γ_{st}				1,4	
Anchorage resistance	γ_a				1,4	

[a]This factor is to be used for tanϕ'. [b] For the analysis of the general stability of slopes and any other structure.

For Eq. (29) parameter b is given by:

$$b = \frac{\sum_{i=1}^{n}(X_i - X_m)(z_i - m_z)}{\sum_{i=1}^{n}(z_i - m_z)^2}. \tag{30}$$

For the 5% fractiles (for the estimation of the extreme value), the error ε_n is given by:

$$\varepsilon_n = t_{n-2}^{95\%} \cdot s_e \cdot \sqrt{(1 + \frac{1}{n}) + \frac{(z - m_z)^2}{\sum_{i=1}^{n}\left[(z_i - m_z)^2\right]}}. \tag{31}$$

And for the 50% fractiles (for the estimation of the average value) by:

$$\varepsilon_n = t_{n-2}^{95\%} \cdot s_e \cdot \sqrt{\frac{1}{n} + \frac{(z - m_z)^2}{\sum_{i=1}^{n}\left[(z_i - m_z)^2\right]}}. \tag{32}$$

The value of the so-called standard error is:

$$s_e = \sqrt{\frac{\sum_{i=1}^{n}\left[(X_i - X_m) - b \cdot (z_i - m_z)\right]^2}{n-2}}, \tag{33}$$

And $t_{n-2}^{95\%}$ is Student's t-value for (n-2) degrees of freedom at the 50% confidence level.

5 The design value

The design values (X_d) of the geotechnical parameters should either be calculated from the characteristic values [1]:

$$X_d = \frac{X_k}{\gamma_M}, \tag{34}$$

where γ_M is the partial factor, or directly estimated (in this case, it is appropriate to regard the values of partial factors suggested in the national enclosures to be authoritative from the aspect of required safety level). For the case of the various ultimate limit states of the load bearing capacity [1], the partial factors should be assumed according to Tab. 2.

The above relationships can be used as a basis for counting back the global confidence level resulting from the design value depending on the partial factors, in case of a characteristic value (lower extreme value) defined at a confidence level of 95%. Tab. 3 and Fig. 6 show this relationship for the case of a sample quantity of $n = 30$ and of statistically "known" parameters. Calculation of one point on the chart in Fig. 6 and in the Tab. 3 can see in Appendix B. The confidence level counted back for the design value in case of partial factors situated between 1,25–1,5 and $v_x < 0,29$ will be higher than 97,5%, in case of $v_x < 0,18$ will exceed 99%, and in case of $v_x < 0,1$ will exceed 99,9%.

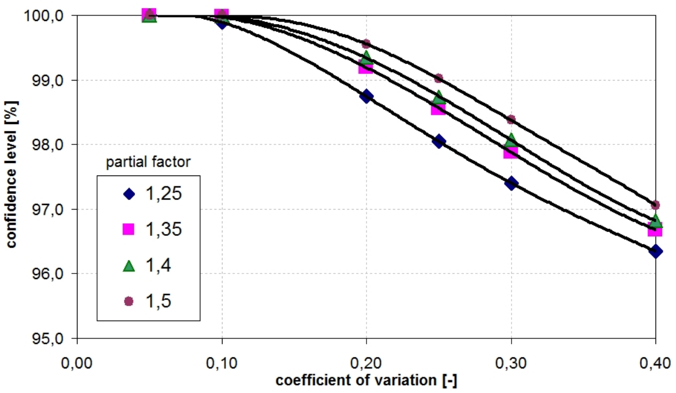

Fig. 6. Global confidence level under consideration of the partial factor and the coefficient of variation (lower extreme value; quantity of samples: n=30; statistically "known" parameters)

6 Conclusions

In geotechnical calculations the definition of soil characteristics represents one of the most critical part of the task, therefore especial care should be taken during the evaluation of the param-

Tab. 3. Numerical values of the results obtained from Fig. 5.

		partial factor (γ_M)			
		1.25	1.35	1.4	1.5
coefficient of variation	0.05	99.9996	99.99998	99.99999	99.99999
	0.10	99.9	99.97	99.98	99.99
	0.20	98.7	99.2	99.3	99.6
	0.25	98.0	98.6	98.7	99.0
	0.30	97.4	97.9	98.1	98.4
	0.40	96.3	96.7	96.8	97.1

eters. This aspect has been further emphasized with the introduction of the unified European regulation system (Eurocode). The methods and procedures was summarized related to the statistical evaluation of the soil parameters, compared the calculation of the characteristic value for the cases of parameters of normal and lognormal distributions, and the mean and the extreme values of the soil parameters are equally considered. The analysis of parameters that vary linearly with depth allows for the extension of statistical calculations operating with multivariables. The comparison of previous knowledges and measurement results plays an important role in geotechnical engineering practice: the possibility of their combination with the described method was presented. Through an uplift analysis performed with a U-shaped reinforced concrete structure, it was demonstrated that it is not clear in each case without the execution of preliminary tests whether the characteristic value is to be interpreted as a lower or upper estimate of the parameter. Finally, the global confidence levels were calculated related to the design values derived from the characteristic value by dividing them with a partial factor. It was found that the confidence level counted back to the design value, in case of a sample quantity of $n = 30$ referring to the extreme value, of statistically "known" parameters, exceeds 97,5% if $\upsilon_x < 0,29$, exceeds 99% if $\upsilon_x < 0,18$, and exceeds 99,9% if $\upsilon_x < 0,1$.

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A Calculation of coefficient k_n for n=10 number of the samples (as in Fig. 3)

Confidence level: $cl = 0,95$ (95%)

Probability : $p = 2 \cdot (1 - cl) = 2 \cdot (1 - 0,95) = 0,1$

The values of statistical parameter k_n (defined in Chapter 2):

Case A-I. (average value, a statistically "known" parameter) (Eq. 3):

$$k_n = t_{\infty}^{95\%}\sqrt{\frac{1}{n}} = 1,645 \cdot \sqrt{\frac{1}{10}} = 0,520$$

Case A-II. (average value, a statistically "unknown" parameter) (Eq. 4):

$$k_n = t_{n-1}^{95\%}\sqrt{\frac{1}{n}} = 1,833 \cdot \sqrt{\frac{1}{10}} = 0,580$$

Case B-I. (extreme value, a "known" parameter) (Eq. 5):

$$k_n = t_{\infty}^{95\%}\sqrt{\frac{1}{n}+1} = 1,645 \cdot \sqrt{\frac{1}{10}+1} = 1,725$$

Case B-II. (extreme value, an "unknown" parameter) (Eq. 6):

$$k_n = t_{n-1}^{95\%}\sqrt{\frac{1}{n}+1} = 1,833 \cdot \sqrt{\frac{1}{10}+1} = 1,923.$$

The values of Student distribution can be calculated by MS-Excel program.

At an infinite degrees of freedom (df=∞), the value of t (by Students' t distribution) at a confidence level of 95 % is as follows (or by Eq. 7):

$t_{\infty}^{95\%}$ =TINV(p;df)=TINV(0,1; ∞) $\approx$1,645.

At df=n-1=9 degrees of freedom, the value of t at a confidence level of 95 % (see on the chart of Fig. 2):

$t_{n-1}^{95\%}$ =TINV(p;df)=TINV(0,1; 9)$\approx$1,833.

TINV (in Hungarian version: INVERZ.T) is a function in MS-Excel and calculates the inverse of the two-tailed Student's t distribution, which is a continuous probability distribution that is frequently used for testing hypotheses on small sample data sets.

The format of the function is: TINV(probability; degrees of freedom)=TINV(p;df)

Where the function arguments are:

- probability (p): the probability (between 0 and 1) for which you want to evaluate the inverse of the Student's t distribution.

- degrees of freedom (df): the number of degrees of freedom (must be $\geq$ 1).

B Calculation of global confidence level for v$_x$=0,25 coefficient of variation and $\gamma_M = 1,35$ partial factor (as in Fig. 6 and Tab. 3.)

number of samples: $n = 30$;

partial factor: $\gamma_M = 1,35$

coefficient of variation: $\upsilon_x = 0,25$

type: lower extreme value; statistically "known" parameters (case B-I.)

The value of statistical parameter k_n in case B-I. ($t_\infty^{95\%} \approx 1,645$ by Eq. 7):

$$k_n = t_\infty^{95\%}\sqrt{\frac{1}{n}+1} = 1,645\cdot\sqrt{\frac{1}{30}+1} = 1,697$$

The characteristic value for case B-I. (as in Chapter 2) (Eq. 1):

$$X_k = X_m\cdot(1-k_n\cdot\nu_x) = X_m\cdot(1-1,697\cdot 0,25) = X_m\cdot 0,576$$

The design value of the parameter (by Eq. 34):

$X_d = \frac{X_k}{\gamma_M} = \frac{X_m\cdot(1-k_n\cdot\nu_x)}{\gamma_M} = \frac{X_m\cdot 0,576}{1,35} = X_m\cdot 0,426$ or the same: $\frac{X_d}{X_m} = 0,426$

The design value by the „new" k_n* value from Eq. (1):

$X_d = X_m\cdot(1-k_n^*\cdot\nu_x)$ or the same: $\frac{X_d}{X_m} = (1-k_n^*\cdot\nu_x)$

So the so-called „new" k_n* value:

$$k_n^* = \left(1-\frac{X_d}{X_m}\right)/\nu_x = (1-0,426)/0,25 = 2,294$$

Probability:

p=TDIST(k_n^*;n;2)=TDIST(2,294;30;2)=0,028949

And the global confidence level:

$cl = 100\cdot(1-p/2) = 100\cdot(1-0,028949/2) \approx 98,6\%$

TDIST (in Hungarian version: T.ELOSZLÁS) is a function in MS-Excel and calculates the Student's t distribution, which is a continuous probability distribution that is frequently used for testing hypotheses on small sample data sets.

The format of the function is: TDIST(X; degrees of freedom; tails)

Where the function arguments are:

– X: the value at which you want to evaluate the Student's t distribution.

– degrees of freedom (df): the number of degrees of freedom (must be ≥ 1)

– tails: the number of distribution tails to return. (This must be either: 1 - to return a one-tailed distribution; 2 - to return a two-tailed distribution).